

Quotient rule

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Practice problems

1. Find the derivative of $f(x) = \frac{x+2}{x-3}$.
2. Find the derivative of $y = \frac{x^2}{2x+1}$.
3. Find the derivative of $g(x) = \frac{e^x}{x^2}$. Decide if the quotient rule or rewriting is more efficient.

Solutions

1. Find the derivative of $f(x) = \frac{x+2}{x-3}$.

- Identify $f(x) = x + 2$ and $g(x) = x - 3$.
- Compute derivatives: $f'(x) = 1$ and $g'(x) = 1$.
- Apply quotient rule: $\frac{(1)(x-3)-(x+2)(1)}{(x-3)^2}$.
- Simplify numerator: $x - 3 - x - 2 = -5$.

Answer: $f'(x) = \frac{-5}{(x-3)^2}$

2. Find the derivative of $y = \frac{x^2}{2x+1}$.

- Identify high as x^2 (derivative $2x$) and low as $2x + 1$ (derivative 2).
- Apply rule: $\frac{(2x)(2x+1)-(x^2)(2)}{(2x+1)^2}$.
- Expand numerator: $4x^2 + 2x - 2x^2$.
- Simplify: $2x^2 + 2x$.

Answer: $y' = \frac{2x^2 + 2x}{(2x+1)^2}$

3. Find the derivative of $g(x) = \frac{e^x}{x^2}$. Decide if the quotient rule or rewriting is more efficient.

- Rewriting as $g(x) = e^x \cdot x^{-2}$ allows the product rule.
- Using product rule: $g'(x) = (e^x)(x^{-2}) + (e^x)(-2x^{-3})$.
- Factor out e^x and x^{-3} : $e^x x^{-3}(x - 2)$.
- Convert back to fraction: $\frac{e^x(x-2)}{x^3}$.

Answer: $g'(x) = \frac{e^x(x-2)}{x^3}$