

Product rule — proof and intuition

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Practice problems

1. Find the derivative of $f(x) = x^3 \ln x$.
2. Find the derivative of $g(x) = x^2 \sin x$.
3. Find the derivative of $h(x) = (x^2 + 1)e^x$.

Solutions

1. Find the derivative of $f(x) = x^3 \ln x$.

- Identify $u = x^3$ and $v = \ln x$.
- Compute derivatives: $u' = 3x^2$ and $v' = \frac{1}{x}$.
- Apply product rule: $f'(x) = (3x^2)(\ln x) + (x^3)(\frac{1}{x})$.
- Simplify the second term: $x^3 \cdot \frac{1}{x} = x^2$.
- Combine: $f'(x) = 3x^2 \ln x + x^2$.

Answer: $f'(x) = x^2(3 \ln x + 1)$

2. Find the derivative of $g(x) = x^2 \sin x$.

- Identify $u = x^2$ and $v = \sin x$.
- Compute derivatives: $u' = 2x$ and $v' = \cos x$.
- Apply product rule: $g'(x) = (2x)(\sin x) + (x^2)(\cos x)$.
- Factor out x : $g'(x) = x(2 \sin x + x \cos x)$.

Answer: $g'(x) = 2x \sin x + x^2 \cos x$

3. Find the derivative of $h(x) = (x^2 + 1)e^x$.

- Identify $u = x^2 + 1$ and $v = e^x$.
- Compute derivatives: $u' = 2x$ and $v' = e^x$.
- Apply product rule: $h'(x) = (2x)(e^x) + (x^2 + 1)(e^x)$.
- Factor out e^x : $h'(x) = e^x(2x + x^2 + 1)$.
- Recognize the quadratic as a perfect square: $x^2 + 2x + 1 = (x + 1)^2$.

Answer: $h'(x) = (x+1)^2 e^x$