

Derivative of x to the n — proof and practice

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Practice problems

1. Find the derivative of $f(x) = x^7$.
2. Find the derivative of $f(x) = \frac{1}{x^4}$.
3. Find the derivative of $f(x) = \sqrt[5]{x^2}$.

Solutions

1. Find the derivative of $f(x) = x^7$.

- Identify the exponent $n = 7$.
- Apply the power rule: bring the 7 to the front and subtract 1 from the exponent.
- The result is $7x^{7-1}$.

Answer: $f'(x) = 7x^6$

2. Find the derivative of $f(x) = \frac{1}{x^4}$.

- Rewrite the function using a negative exponent: $f(x) = x^{-4}$.
- Apply the power rule: bring -4 to the front and subtract 1 from the exponent.
- Compute $-4 - 1 = -5$.
- The derivative is $-4x^{-5}$, which can be rewritten as $-\frac{4}{x^5}$.

Answer: $f'(x) = -\frac{4}{x^5}$

3. Find the derivative of $f(x) = \sqrt[5]{x^2}$.

- Rewrite the radical as a fractional exponent: $f(x) = (x^2)^{1/5} = x^{2/5}$.
- Apply the power rule: bring $2/5$ to the front and subtract 1 from the exponent.
- Compute $2/5 - 1 = 2/5 - 5/5 = -3/5$.
- The derivative is $\frac{2}{5}x^{-3/5}$.

Answer: $f'(x) = \frac{2}{5}x^{-3/5}$