The derivative as instantaneous rate of change

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Practice problems

1. A particle moves along a line with position $s(t) = 3t^2 - 2t$, where s is in meters and t is in seconds. Find the instantaneous velocity at $t = 1$.
2. The cost of producing x units of a product is given by $C(x) = 0.5x^2 + 10x + 100$ dollars. Find the marginal cost (the instantaneous rate of change of cost) when $x = 10$.
3. An object's position is given by $s(t) = 2t^2$. Determine the time $t > 0$ at which the instantaneous velocity is exactly 12 m/s.

Solutions

1. A particle moves along a line with position $s(t) = 3t^2 - 2t$, where s is in meters and t is in seconds. Find the instantaneous velocity at $t = 1$.

   - Set up the limit: $v(1) = \lim_{h\to 0} \frac{s(1+h)-s(1)}{h}$
   - Calculate $s(1) = 3(1)^2 - 2(1) = 1$
   - Calculate $s(1+h) = 3(1+h)^2 - 2(1+h) = 3(1+2h+h^2) - 2 - 2h = 3+6h+3h^2-2-2h = 1 + 4h + 3h^2$
   - Substitute into the limit: $\lim_{h\to 0} \frac{(1+4h+3h^2)-1}{h} = \lim_{h\to 0} \frac{4h+3h^2}{h}$
   - Simplify: $\lim_{h\to 0}(4 + 3h) = 4$

   Answer: 4 \text m/s

2. The cost of producing x units of a product is given by $C(x) = 0.5x^2 + 10x + 100$ dollars. Find the marginal cost (the instantaneous rate of change of cost) when $x = 10$.

   - Set up the limit for the derivative at $x = 10$: $C'(10) = \lim_{h\to 0} \frac{C(10+h)-C(10)}{h}$
   - Calculate $C(10) = 0.5(100) + 10(10) + 100 = 50 + 100 + 100 = 250$
   - Calculate $C(10+h) = 0.5(10+h)^2+10(10+h)+100 = 0.5(100+20h+h^2)+100+10h+100 = 50 + 10h + 0.5h^2 + 200 + 10h = 250 + 20h + 0.5h^2$
   - Substitute into the limit: $\lim_{h\to 0} \frac{(250+20h+0.5h^2)-250}{h} = \lim_{h\to 0} \frac{20h+0.5h^2}{h}$
   - Simplify: $\lim_{h\to 0}(20 + 0.5h) = 20$

   Answer: 20 \text dollars per unit

3. An object's position is given by $s(t) = 2t^2$. Determine the time $t > 0$ at which the instantaneous velocity is exactly 12 m/s.

   - Find the general derivative $s'(t)$: $\lim_{h\to 0} \frac{2(t+h)^2-2t^2}{h}$
   - Expand $2(t + h)^2 = 2(t^2 + 2th + h^2) = 2t^2 + 4th + 2h^2$
   - Simplify the limit: $\lim_{h\to 0} \frac{4th+2h^2}{h} = \lim_{h\to 0}(4t + 2h) = 4t$
   - Set the velocity equal to 12: $4t = 12$
   - Solve for t: $t = 3$

   Answer: t = 3 \text seconds
