The derivative as a limit: slope of the tangent

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Practice problems

1. Find the derivative of $f(x) = x^2 + 2x$ at $a = 1$ using the limit definition.
2. Find the slope of the tangent line to $f(x) = x^2 - 3x$ at $a = 2$.
3. Determine the derivative of $f(x) = 3x^2 - 5$ at $a = -1$.

Solutions

1. Find the derivative of $f(x) = x^2 + 2x$ at $a = 1$ using the limit definition.

   - Set up the limit: $\lim_{h\to 0} \frac{(1+h)^2+2(1+h)-(1^2+2(1))}{h}$
   - Expand the numerator: $\lim_{h\to 0} \frac{1+2h+h^2+2+2h-3}{h}$
   - Simplify the numerator: $\lim_{h\to 0} \frac{4h+h^2}{h}$
   - Factor and cancel h: $\lim_{h\to 0}(4+h)$
   - Evaluate the limit as $h \to 0$: $4 + 0 = 4$

   Answer: 4

2. Find the slope of the tangent line to $f(x) = x^2 - 3x$ at $a = 2$.

   - Set up the limit: $\lim_{h\to 0} \frac{((2+h)^2-3(2+h))-(2^2-3(2))}{h}$
   - Expand the terms: $\lim_{h\to 0} \frac{4+4h+h^2-6-3h-(4-6)}{h}$
   - Simplify the numerator: $\lim_{h\to 0} \frac{h^2+h-2-(-2)}{h} = \lim_{h\to 0} \frac{h^2+h}{h}$
   - Factor and cancel h: $\lim_{h\to 0}(h+1)$
   - Evaluate the limit: $0 + 1 = 1$

   Answer: 1

3. Determine the derivative of $f(x) = 3x^2 - 5$ at $a = -1$.

   - Set up the limit: $\lim_{h\to 0} \frac{3(-1+h)^2-5-(3(-1)^2-5)}{h}$
   - Expand the squared term: $\lim_{h\to 0} \frac{3(1-2h+h^2)-5-(3-5)}{h}$
   - Simplify the numerator: $\lim_{h\to 0} \frac{3-6h+3h^2-5-(-2)}{h}$
   - Combine constants: $\lim_{h\to 0} \frac{-6h+3h^2}{h}$
   - Factor and cancel h: $\lim_{h\to 0}(-6+3h)$
   - Evaluate the limit: -6

   Answer: -6
