Module 1 review + self-test

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Practice problems

1. Evaluate the limit: $\lim_{x \to 4} \frac{x^2 - 16}{x - 4}$
2. Evaluate the limit: $\lim_{x \to 0} \frac{\sin(7x)}{2x}$
3. Find the horizontal asymptote of $f(x) = \frac{3x^2 - 5}{2x^2 + x + 1}$ by computing the limit as $x \to \infty$.

Solutions

1. Evaluate the limit: $\lim_{x\to 4} \frac{x^2-16}{x-4}$

   - Substitute $x = 4$ to get $\frac{16-16}{4-4} = \frac{0}{0}$, an indeterminate form.
   - Factor the numerator as a difference of squares: $x^2 - 16 = (x-4)(x+4)$.
   - Cancel the common factor $(x-4)$ from the top and bottom.
   - Evaluate the remaining limit: $\lim_{x\to 4}(x+4) = 4+4 = 8$.

   Answer: 8

2. Evaluate the limit: $\lim_{x\to 0} \frac{\sin(7x)}{2x}$

   - Identify the form $\frac{\sin(ax)}{bx}$.
   - Rewrite the expression to isolate the $\frac{\sin(7x)}{7x}$ form: $\frac{7}{2} \cdot \frac{\sin(7x)}{7x}$.
   - Apply the special limit $\lim_{u\to 0} \frac{\sin u}{u} = 1$ where $u = 7x$.
   - The result is $\frac{7}{2} \cdot 1 = 3.5$.

   Answer: 7/2

3. Find the horizontal asymptote of $f(x) = \frac{3x^2-5}{2x^2+x+1}$ by computing the limit as $x \to \infty$.

   - Identify the highest power of x in the denominator, which is x^2.
   - Divide every term by x^2: $\frac{3-5/x^2}{2+1/x+1/x^2}$.
   - Take the limit as $x \to \infty$, noting that $5/x^2$, $1/x$, and $1/x^2$ all approach 0.
   - The limit is $\frac{3-0}{2+0+0} = \frac{3}{2}$.

   Answer: y = 3/2
