Common limit pitfalls and how to avoid them

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Practice problems

1. Evaluate the limit: $\lim_{x\to -2} \frac{x^2+5x+6}{x+2}$
2. Evaluate the limit: $\lim_{x\to \infty} \frac{3x^2-2x+1}{5x^2+7}$
3. Evaluate the limit: $\lim_{x\to 0} \frac{|x|}{x^2+x}$

Solutions

1. Evaluate the limit: $\lim_{x\to -2} \frac{x^2+5x+6}{x+2}$

   - Plugging in $x = -2$ gives $\frac{(-2)^2+5(-2)+6}{-2+2} = \frac{4-10+6}{0} = \frac{0}{0}$, which is an indeterminate form.
   - Factor the numerator: $x^2 + 5x + 6 = (x + 2)(x + 3)$.
   - Rewrite the limit: $\lim_{x\to -2} \frac{(x+2)(x+3)}{x+2}$.
   - Cancel the common factor $(x + 2)$ from the top and bottom: $\lim_{x\to -2}(x + 3)$.
   - Substitute $x = -2$ into the simplified expression: $-2 + 3 = 1$.

   Answer: 1

2. Evaluate the limit: $\lim_{x\to\infty} \frac{3x^2-2x+1}{5x^2+7}$

   - Identify the highest power of x in the denominator, which is x^2.
   - Divide every term by x^2: $\lim_{x\to\infty} \frac{3-\frac{2}{x}+\frac{1}{x^2}}{5+\frac{7}{x^2}}$.
   - Apply the limit laws: $\frac{2}{x}$, $\frac{1}{x^2}$, and $\frac{7}{x^2}$ all approach 0 as x approaches infinity.
   - The expression becomes $\frac{3-0+0}{5+0} = \frac{3}{5}$.

   Answer: \frac35

3. Evaluate the limit: $\lim_{x\to 0} \frac{|x|}{x^2+x}$

   - Check the limit from the right $(x > 0)$: $\lim_{x\to 0^+} \frac{x}{x(x+1)} = \lim_{x\to 0^+} \frac{1}{x+1} = 1$.
   - Check the limit from the left $(x < 0)$: $\lim_{x\to 0^-} \frac{-x}{x(x+1)} = \lim_{x\to 0^-} \frac{-1}{x+1} = -1$.
   - Since the left-hand limit (-1) and right-hand limit (1) are not equal, the two-sided limit does not exist.

   Answer: \textDoes Not Exist
