Limits of trigonometric combinations — technique clinic

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Practice problems

1. Evaluate the limit: $\lim_{x\to 0} \frac{\sin(7x)}{2x}$
2. Evaluate the limit: $\lim_{x\to 0} \frac{1-\cos(4x)}{x}$
3. Evaluate the limit: $\lim_{x\to 0} \frac{\sin(3x)+1-\cos(x)}{x}$

Solutions

1. Evaluate the limit: $\lim_{x\to 0} \frac{\sin(7x)}{2x}$

   - Identify that the argument of sine is $7x$ but the denominator is $2x$.
   - Multiply the numerator and denominator by 7 to match the argument: $\frac{7}{2} \cdot \frac{\sin(7x)}{7x}$.
   - Apply the limit $\lim_{u\to 0} \frac{\sin(u)}{u} = 1$ where $u = 7x$.
   - The result is $\frac{7}{2} \cdot 1 = 3.5$.

   Answer: 3.5

2. Evaluate the limit: $\lim_{x\to 0} \frac{1-\cos(4x)}{x}$

   - Identify that the argument of cosine is $4x$.
   - To use the formula $\lim_{u\to 0} \frac{1-\cos(u)}{u} = 0$, we need $4x$ in the denominator.
   - Multiply the top and bottom by 4: $4 \cdot \frac{1-\cos(4x)}{4x}$.
   - The limit of the fraction part is 0, so $4 \cdot 0 = 0$.

   Answer: 0

3. Evaluate the limit: $\lim_{x\to 0} \frac{\sin(3x)+1-\cos(x)}{x}$

   - Split the limit into two separate fractions: $\lim_{x\to 0} \frac{\sin(3x)}{x} + \lim_{x\to 0} \frac{1-\cos(x)}{x}$.
   - For the first part, multiply by 3/3 to get $3 \cdot \lim_{x\to 0} \frac{\sin(3x)}{3x} = 3 \cdot 1 = 3$.
   - For the second part, use the known limit $\lim_{x\to 0} \frac{1-\cos(x)}{x} = 0$.
   - Add the results: $3 + 0 = 3$.

   Answer: 3
