

Famous limit: $(1 + 1/n)^n$ approaches e

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Practice problems

1. Evaluate the limit: $\lim_{n \rightarrow \infty} (1 + \frac{3}{n})^n$
2. Evaluate the limit: $\lim_{n \rightarrow \infty} (1 + \frac{1}{2n})^{2n}$
3. Evaluate the limit: $\lim_{n \rightarrow \infty} (1 - \frac{2}{n})^n$

Solutions

1. Evaluate the limit: $\lim_{n \rightarrow \infty} (1 + \frac{3}{n})^n$

- Let $m = \frac{n}{3}$, which implies $n = 3m$.
- As $n \rightarrow \infty$, $m \rightarrow \infty$.
- Rewrite the limit as $\lim_{m \rightarrow \infty} (1 + \frac{1}{m})^{3m}$.
- Apply the power rule for limits: $[\lim_{m \rightarrow \infty} (1 + \frac{1}{m})^m]^3$.
- Substitute the definition of e : e^3 .

Answer: e^3

2. Evaluate the limit: $\lim_{n \rightarrow \infty} (1 + \frac{1}{2n})^{2n}$

- Let $m = 2n$.
- As $n \rightarrow \infty$, $m \rightarrow \infty$.
- Substitute m into the expression: $\lim_{m \rightarrow \infty} (1 + \frac{1}{m})^m$.
- This is the exact definition of e .

Answer: e

3. Evaluate the limit: $\lim_{n \rightarrow \infty} (1 - \frac{2}{n})^n$

- Rewrite the expression as $(1 + \frac{-2}{n})^n$.
- Let $m = \frac{n}{-2}$, so $n = -2m$.
- As $n \rightarrow \infty$, $m \rightarrow -\infty$.
- The limit becomes $\lim_{m \rightarrow -\infty} (1 + \frac{1}{m})^{-2m}$.
- This is $[\lim_{m \rightarrow -\infty} (1 + \frac{1}{m})^m]^{-2}$.
- Substitute e : e^{-2} or $\frac{1}{e^2}$.

Answer: e^{-2}