Epsilon-delta worked examples

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Practice problems

1. Prove using the epsilon-delta definition that $\lim_{x\to 4}(3x+2) = 14$.
2. Prove using the epsilon-delta definition that $\lim_{x\to 3} x^2 = 9$.
3. Prove using the epsilon-delta definition that $\lim_{x\to 1}(x^2+2x) = 3$.

Solutions

1. Prove using the epsilon-delta definition that $\lim_{x\to 4}(3x+2) = 14$.

   - Start with $|(3x+2) - 14| < \epsilon$.
   - Simplify to $|3x - 12| < \epsilon$, which is $3|x-4| < \epsilon$.
   - This suggests $|x-4| < \epsilon/3$.
   - Choose $\delta = \epsilon/3$.
   - Verify: if $|x-4| < \delta$, then $3|x-4| < 3(\epsilon/3) = \epsilon$, so $|(3x+2) - 14| < \epsilon$.

   Answer: \delta = \frac\epsilon3

2. Prove using the epsilon-delta definition that $\lim_{x\to 3} x^2 = 9$.

   - Start with $|x^2 - 9| = |x-3||x+3| < \epsilon$.
   - Assume $|x-3| < 1$, so $2 < x < 4$.
   - Then $|x+3| < 4 + 3 = 7$.
   - We need $|x-3| \cdot 7 < \epsilon$, so $|x-3| < \epsilon/7$.
   - Choose $\delta = \min(1, \epsilon/7)$.

   Answer: \delta = \min(1, \frac\epsilon7)

3. Prove using the epsilon-delta definition that $\lim_{x\to 1}(x^2 + 2x) = 3$.

   - Start with $|(x^2 + 2x) - 3| = |(x-1)(x+3)| < \epsilon$.
   - Assume $|x-1| < 1$, so $0 < x < 2$.
   - Then $|x+3| < 2 + 3 = 5$.
   - We need $|x-1| \cdot 5 < \epsilon$, so $|x-1| < \epsilon/5$.
   - Choose $\delta = \min(1, \epsilon/5)$.

   Answer: \delta = \min(1, \frac\epsilon5)
