The formal epsilon-delta definition, gently

Practice problems

1. Use the formal $\epsilon - \delta$ definition to prove that $\lim_{x \to 3}(4x - 5) = 7$.
2. Prove that $\lim_{x \to -1}(2x + 8) = 6$ using the formal definition.
3. Given the function $f(x) = -5x + 2$, find the formula for δ in terms of ϵ that proves $\lim_{x \to a} f(x) = L$ for any a and L.

Solutions

1. Use the formal $\epsilon - \delta$ definition to prove that $\lim_{x \to 3}(4x - 5) = 7$.

   - Start with the output inequality: $|(4x - 5) - 7| < \epsilon$.
   - Simplify the expression: $|4x - 12| < \epsilon$.
   - Factor out the constant: $4|x - 3| < \epsilon$.
   - Isolate the input distance: $|x - 3| < \frac{\epsilon}{4}$.
   - Choose $\delta = \frac{\epsilon}{4}$. If $0 < |x - 3| < \delta$, then $4|x - 3| < \epsilon$, which implies $|(4x - 5) - 7| < \epsilon$.

   Answer: \delta = \frac\epsilon4

2. Prove that $\lim_{x \to -1}(2x + 8) = 6$ using the formal definition.

   - Set up the inequality: $|(2x + 8) - 6| < \epsilon$.
   - Simplify: $|2x + 2| < \epsilon$.
   - Factor: $2|x + 1| < \epsilon$.
   - Solve for the distance from x to -1: $|x - (-1)| < \frac{\epsilon}{2}$.
   - Choose $\delta = \frac{\epsilon}{2}$.

   Answer: \delta = \frac\epsilon2

3. Given the function $f(x) = -5x+2$, find the formula for δ in terms of ϵ that proves $\lim_{x \to a} f(x) = L$ for any a and L.

   - Start with $|(-5x + 2) - L| < \epsilon$.
   - Since $L = -5a + 2$, substitute this in: $|-5x + 2 - (-5a + 2)| < \epsilon$.
   - Simplify: $|-5x + 5a| < \epsilon$.
   - Factor out -5: $|-5(x - a)| < \epsilon$.
   - Use absolute value properties: $5|x - a| < \epsilon$.
   - Divide by 5: $|x - a| < \frac{\epsilon}{5}$.

   Answer: \delta = \frac\epsilon5
