Limits at infinity and horizontal asymptotes

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Practice problems

1. Find the limit as x approaches infinity of the function $f(x) = \frac{6x^2-3x+1}{2x^2+5}$.
2. Determine the horizontal asymptote(s) of the function $g(x) = \frac{5x+2}{x^2-9}$.
3. Evaluate the limit as x approaches infinity of $h(x) = \frac{2x^3-4x}{x^3+7x^2+1}$.

Solutions

1. Find the limit as x approaches infinity of the function $f(x) = \frac{6x^2-3x+1}{2x^2+5}$.

   - Identify the highest power of x in the denominator, which is x^2.
   - Divide every term in the numerator and denominator by x^2: $\frac{6-3/x+1/x^2}{2+5/x^2}$.
   - Apply the limit as $x \to \infty$, noting that $3/x$, $1/x^2$, and $5/x^2$ all approach 0.
   - The expression simplifies to $6/2 = 3$.

   Answer: 3

2. Determine the horizontal asymptote(s) of the function $g(x) = \frac{5x+2}{x^2-9}$.

   - Compare the degrees of the numerator and denominator. The numerator degree is 1 and the denominator degree is 2.
   - Since the degree of the denominator is higher, the limit as $x \to \infty$ is 0.
   - Similarly, the limit as $x \to -\infty$ is 0.
   - The horizontal asymptote is the line $y = 0$.

   Answer: y = 0

3. Evaluate the limit as x approaches infinity of $h(x) = \frac{2x^3-4x}{x^3+7x^2+1}$.

   - Divide all terms by the highest power in the denominator, x^3.
   - The expression becomes $\frac{2-4/x^2}{1+7/x+1/x^3}$.
   - As $x \to \infty$, the terms $4/x^2$, $7/x$, and $1/x^3$ all approach 0.
   - The limit is $2/1 = 2$.

   Answer: 2
