

Limit laws and evaluating limits algebraically

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Practice problems

1. Evaluate the limit: $\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5}$
2. Evaluate the limit: $\lim_{x \rightarrow 2} (4x^3 - 2x + 7)$
3. Evaluate the limit: $\lim_{x \rightarrow 1} \frac{x^2 + 2x - 3}{x^2 - 1}$

Solutions

1. Evaluate the limit: $\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5}$

- Substitute $x = 5$ to check for indeterminate form: $\frac{5^2 - 25}{5 - 5} = \frac{0}{0}$.
- Factor the numerator as a difference of squares: $x^2 - 25 = (x - 5)(x + 5)$.
- Cancel the common factor $(x - 5)$ from the top and bottom: $\lim_{x \rightarrow 5} (x + 5)$.
- Use direct substitution: $5 + 5 = 10$.

Answer: 10

2. Evaluate the limit: $\lim_{x \rightarrow 2} (4x^3 - 2x + 7)$

- Recognize that the function is a polynomial, so we can use direct substitution.
- Substitute $x = 2$ into the expression: $4(2)^3 - 2(2) + 7$.
- Calculate the powers: $4(8) - 4 + 7$.
- Perform the multiplication and addition: $32 - 4 + 7 = 35$.

Answer: 35

3. Evaluate the limit: $\lim_{x \rightarrow 1} \frac{x^2 + 2x - 3}{x^2 - 1}$

- Substitute $x = 1$ to check: $\frac{1 + 2 - 3}{1 - 1} = \frac{0}{0}$.
- Factor the numerator: $x^2 + 2x - 3 = (x - 1)(x + 3)$.
- Factor the denominator: $x^2 - 1 = (x - 1)(x + 1)$.
- Cancel the common factor $(x - 1)$: $\lim_{x \rightarrow 1} \frac{x + 3}{x + 1}$.
- Substitute $x = 1$: $\frac{1 + 3}{1 + 1} = \frac{4}{2} = 2$.

Answer: 2