One-sided limits; when limits fail to exist

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Practice problems

1. Find the limit $\lim_{x \to -1} f(x)$ for the piecewise function $f(x) = x^2$ if $x < -1$ and $f(x) = -x$ if $x \geq -1$.
2. Determine if the limit $\lim_{x \to 2} \frac{1}{(x-2)^2}$ exists. If not, explain why.
3. Consider the function $f(x) = \frac{|x+3|}{x+3}$. Find $\lim_{x \to -3} f(x)$.

Solutions

1. Find the limit $\lim_{x \to -1} f(x)$ for the piecewise function $f(x) = x^2$ if $x < -1$ and $f(x) = -x$ if $x \geq -1$.

   - Compute the left-hand limit: $\lim_{x \to -1^-} x^2 = (-1)^2 = 1$.
   - Compute the right-hand limit: $\lim_{x \to -1^+} (-x) = -(-1) = 1$.
   - Since the left-hand limit equals the right-hand limit, the overall limit exists.

   Answer: 1

2. Determine if the limit $\lim_{x \to 2} \frac{1}{(x-2)^2}$ exists. If not, explain why.

   - As x approaches 2 from the right, $(x-2)^2$ is a very small positive number, so $1/(x-2)^2$ goes to ∞.
   - As x approaches 2 from the left, $(x-2)^2$ is also a very small positive number, so $1/(x-2)^2$ goes to ∞.
   - Because the function is unbounded, it does not approach a finite real number.

   Answer: DNE (Unbounded)

3. Consider the function $f(x) = \frac{|x+3|}{x+3}$. Find $\lim_{x \to -3} f(x)$.

   - For $x > -3$, $|x+3| = x+3$, so $f(x) = (x+3)/(x+3) = 1$. The right-hand limit is 1.
   - For $x < -3$, $|x+3| = -(x+3)$, so $f(x) = -(x+3)/(x+3) = -1$. The left-hand limit is -1.
   - Since $1 \neq -1$, the limit does not exist.

   Answer: DNE (Jump)
