The limit concept: numerical and graphical intuition

Lesson 13 · Module 1 — Limits & Continuity · The Calculus Course · dailymaths.uk/course

Practice problems

1. Use a table of values to estimate the limit: $\lim_{x\to 3} \frac{x^2-9}{x-3}$.
2. Given the piecewise function $f(x) = x+5$ for $x < 1$ and $f(x) = 2x+4$ for $x \geq 1$, find $\lim_{x\to 1} f(x)$.
3. Determine if the limit $\lim_{x\to 0} \frac{1}{x}$ exists. Explain using one-sided limits.

Solutions

1. Use a table of values to estimate the limit: $\lim_{x\to 3} \frac{x^2-9}{x-3}$.

   - Pick values approaching 3 from the left: $x = 2.9, 2.99, 2.999$.
   - Calculate $f(2.9) = \frac{8.41-9}{2.9-3} = \frac{-0.59}{-0.1} = 5.9$.
   - Calculate $f(2.99) = \frac{8.9401-9}{2.99-3} = \frac{-0.0599}{-0.01} = 5.99$.
   - Pick values approaching 3 from the right: $x = 3.1, 3.01, 3.001$.
   - Calculate $f(3.1) = \frac{9.61-9}{3.1-3} = \frac{0.61}{0.1} = 6.1$.
   - Calculate $f(3.01) = \frac{9.0601-9}{3.01-3} = \frac{0.0601}{0.01} = 6.01$.
   - Observe that both sides approach 6.

   Answer: 6

2. Given the piecewise function $f(x) = x+5$ for $x < 1$ and $f(x) = 2x+4$ for $x \geq 1$, find $\lim_{x\to 1} f(x)$.

   - Find the left-hand limit: $\lim_{x\to 1^-}(x + 5) = 1 + 5 = 6$.
   - Find the right-hand limit: $\lim_{x\to 1^+}(2x + 4) = 2(1) + 4 = 6$.
   - Since the left-hand limit equals the right-hand limit, the general limit exists.

   Answer: 6

3. Determine if the limit $\lim_{x\to 0} \frac{1}{x}$ exists. Explain using one-sided limits.

   - Approach 0 from the right: as $x \to 0^+$, $1/x$ becomes very large, so $\lim_{x\to 0^+} \frac{1}{x} = \infty$.
   - Approach 0 from the left: as $x \to 0^-$, $1/x$ becomes very large and negative, so $\lim_{x\to 0^-} \frac{1}{x} = -\infty$.
   - Since the left-hand and right-hand limits are not equal (and are not finite), the limit does not exist.

   Answer: Does Not Exist
