The idea of 'approaching' — an informal taste of limits

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Practice problems

1. Find the value that $f(x) = x^2 + 2$ approaches as x approaches 1.
2. Determine the limit of $f(x) = \frac{x^2-16}{x-4}$ as x approaches 4.
3. Consider the function $f(x) = 5$ for $x < 2$ and $f(x) = 10$ for $x \geq 2$. Does the limit exist as x approaches 2? Explain why or why not.

Solutions

1. Find the value that $f(x) = x^2 + 2$ approaches as x approaches 1.

   - Step 1: Plug in values close to 1 from the left, such as 0.9 and 0.99. $f(0.9) = 0.81+2 = 2.81$ and $f(0.99) = 0.9801 + 2 = 2.9801$.
   - Step 2: Plug in values close to 1 from the right, such as 1.1 and 1.01. $f(1.1) = 1.21+2 = 3.21$ and $f(1.01) = 1.0201 + 2 = 3.0201$.
   - Step 3: Observe that both sides are approaching the value 3.

   Answer: 3

2. Determine the limit of $f(x) = \frac{x^2-16}{x-4}$ as x approaches 4.

   - Step 1: Notice that plugging in $x = 4$ results in 0/0, which is undefined.
   - Step 2: Factor the numerator as a difference of squares: $x^2 - 16 = (x - 4)(x + 4)$.
   - Step 3: Simplify the function: $f(x) = \frac{(x-4)(x+4)}{x-4} = x + 4$ for $x \neq 4$.
   - Step 4: As x approaches 4, the expression $x + 4$ approaches $4 + 4 = 8$.

   Answer: 8

3. Consider the function $f(x) = 5$ for $x < 2$ and $f(x) = 10$ for $x \geq 2$. Does the limit exist as x approaches 2? Explain why or why not.

   - Step 1: Approach $x = 2$ from the left. For all $x < 2$, $f(x) = 5$, so the left-hand limit is 5.
   - Step 2: Approach $x = 2$ from the right. For all $x \geq 2$, $f(x) = 10$, so the right-hand limit is 10.
   - Step 3: Compare the two sides. Since $5 \neq 10$, the function does not approach a single value.

   Answer: No, the limit does not exist because the left-hand and right-hand limits are different.
