Composite functions and transformations

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Practice problems

1. Given $f(x) = x^2$ and $g(x) = 3x - 2$, find the expression for $f(g(x))$ and simplify it.
2. Let $f(x) = \sin(x)$. Describe the transformations required to obtain the graph of $h(x) = 3\sin(x - \frac{\pi}{4}) + 1$.
3. Given $f(x) = \frac{1}{x}$ and $g(x) = \sin(x)$, find the domain of $f(g(x))$ for x in the interval $[0, \pi]$.

Solutions

1. Given $f(x) = x^2$ and $g(x) = 3x - 2$, find the expression for $f(g(x))$ and simplify it.

   - Identify the inner function $g(x) = 3x - 2$.
   - Substitute $g(x)$ into $f(x)$: $f(g(x)) = (3x - 2)^2$.
   - Expand the square: $(3x)^2 - 2(3x)(2) + 2^2$.
   - Simplify the terms: $9x^2 - 12x + 4$.

   Answer: $f(g(x)) = 9x^2 - 12x + 4$

2. Let $f(x) = \sin(x)$. Describe the transformations required to obtain the graph of $h(x) = 3\sin(x - \frac{\pi}{4}) + 1$.

   - The multiplier 3 outside the function indicates a vertical stretch by a factor of 3.
   - The term $-\frac{\pi}{4}$ inside the function indicates a horizontal shift to the right by $\frac{\pi}{4}$ units.
   - The constant $+1$ at the end indicates a vertical shift upwards by 1 unit.

   Answer: Vertical stretch by 3, horizontal shift right by $\frac{\pi}{4}$, and vertical shift up by 1.

3. Given $f(x) = \frac{1}{x}$ and $g(x) = \sin(x)$, find the domain of $f(g(x))$ for x in the interval $[0, \pi]$.

   - The composite function is $f(g(x)) = \frac{1}{\sin(x)}$.
   - The inner function $g(x) = \sin(x)$ is defined for all real numbers.
   - The outer function $f(u) = \frac{1}{u}$ is undefined where $u = 0$. Thus, we need $\sin(x) \neq 0$.
   - In the interval $[0, \pi]$, $\sin(x) = 0$ at $x = 0$ and $x = \pi$.

   Answer: $x \neq 0, \pi$
