Trigonometric functions on the unit circle

Practice problems

1. Find the exact values of $\sin(\frac{5\pi}{6})$, $\cos(\frac{5\pi}{6})$, and $\tan(\frac{5\pi}{6})$.
2. Convert $225°$ to radians and find the value of $\cos(225°)$.
3. If $\sin(\theta) = \frac{3}{5}$ and θ is in the second quadrant, find $\cos(\theta)$ and $\tan(\theta)$.

Solutions

1. Find the exact values of $\sin(\frac{5\pi}{6})$, $\cos(\frac{5\pi}{6})$, and $\tan(\frac{5\pi}{6})$.

   - The angle $\frac{5\pi}{6}$ is in the second quadrant because it is between $\frac{\pi}{2}$ and π.
   - The reference angle is $\pi - \frac{5\pi}{6} = \frac{\pi}{6}$ (or $30°$).
   - For $\frac{\pi}{6}$, the coordinates are $(\frac{\sqrt{3}}{2}, \frac{1}{2})$.
   - In the second quadrant, x is negative and y is positive, so the point is $(-\frac{\sqrt{3}}{2}, \frac{1}{2})$.
   - Therefore, $\sin(\frac{5\pi}{6}) = \frac{1}{2}$, $\cos(\frac{5\pi}{6}) = -\frac{\sqrt{3}}{2}$, and $\tan(\frac{5\pi}{6}) = \frac{1/2}{-\sqrt{3}/2} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$.

   Answer: \sin(\frac5\pi6) = \frac12, \cos(\frac5\pi6) = -\frac\sqrt32, \tan(\frac5\pi6) = -\frac\sqrt33

2. Convert $225°$ to radians and find the value of $\cos(225°)$.

   - To convert to radians, multiply by $\frac{\pi}{180}$: $225 \times \frac{\pi}{180} = \frac{225\pi}{180} = \frac{5\pi}{4}$.
   - The angle $\frac{5\pi}{4}$ is in the third quadrant.
   - The reference angle is $\frac{5\pi}{4} - \pi = \frac{\pi}{4}$.
   - The cosine of $\frac{\pi}{4}$ is $\frac{\sqrt{2}}{2}$.
   - In the third quadrant, cosine is negative, so $\cos(225°) = -\frac{\sqrt{2}}{2}$.

   Answer: \frac5\pi4 \text radians, \cos(225^\circ) = -\frac\sqrt22

3. If $\sin(\theta) = \frac{3}{5}$ and θ is in the second quadrant, find $\cos(\theta)$ and $\tan(\theta)$.

   - Use the identity $\sin^2(\theta) + \cos^2(\theta) = 1$.
   - Substitute the known value: $(\frac{3}{5})^2 + \cos^2(\theta) = 1 \implies \frac{9}{25} + \cos^2(\theta) = 1$.
   - Solve for $\cos^2(\theta)$: $\cos^2(\theta) = 1 - \frac{9}{25} = \frac{16}{25}$.
   - Take the square root: $\cos(\theta) = \pm\frac{4}{5}$.
   - Since θ is in the second quadrant, $\cos(\theta)$ must be negative, so $\cos(\theta) = -\frac{4}{5}$.
   - Find tangent: $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{3/5}{-4/5} = -\frac{3}{4}$.

   Answer: \cos(\theta) = -\frac45, \tan(\theta) = -\frac34
