Logarithms as inverses

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Practice problems

1. Solve for x in the equation $3^x = 20$. Give your answer in terms of natural logarithms.
2. Simplify the expression $\log_5(125 \cdot 5^2)$ into a single integer.
3. Solve for x in the equation $e^{2x} = 7$.

Solutions

1. Solve for x in the equation $3^x = 20$. Give your answer in terms of natural logarithms.

   - Take the natural logarithm of both sides: $\ln(3^x) = \ln(20)$
   - Apply the power law to bring x to the front: $x\ln(3) = \ln(20)$
   - Divide both sides by $\ln(3)$ to isolate x: $x = \frac{\ln(20)}{\ln(3)}$

   Answer: $x = \frac{\ln(20)}{\ln(3)}$

2. Simplify the expression $\log_5(125 \cdot 5^2)$ into a single integer.

   - Use the product law: $\log_5(125) + \log_5(5^2)$
   - Evaluate $\log_5(125)$ since $5^3 = 125$, so $\log_5(125) = 3$
   - Use the power law for the second term: $2\log_5(5) = 2 \cdot 1 = 2$
   - Add the results: $3 + 2 = 5$

   Answer: 5

3. Solve for x in the equation $e^{2x} = 7$.

   - Take the natural logarithm of both sides: $\ln(e^{2x}) = \ln(7)$
   - Since ln and e are inverses, $\ln(e^{2x}) = 2x$
   - The equation becomes $2x = \ln(7)$
   - Divide by 2: $x = \frac{\ln(7)}{2}$

   Answer: $x = \frac{\ln(7)}{2}$
