Polynomials and power functions

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Practice problems

1. Find the roots and describe the end behavior of $f(x) = x^2 - 5x + 6$.
2. Find the roots and describe the end behavior of $f(x) = x^3 - 5x^2 + 6x$.
3. Does the polynomial $f(x) = x^3 + x$ have any roots other than $x = 0$? Explain using factorisation.

Solutions

1. Find the roots and describe the end behavior of $f(x) = x^2 - 5x + 6$.

   - Set the function to zero: $x^2 - 5x + 6 = 0$.
   - Factorise the quadratic: $(x - 2)(x - 3) = 0$.
   - Solve for x: $x = 2$ and $x = 3$.
   - Identify the degree as 2 (even) and the leading coefficient as 1 (positive), so both ends point upwards.

   Answer: Roots: $x = 2, 3$; End behavior: both ends point up.

2. Find the roots and describe the end behavior of $f(x) = x^3 - 5x^2 + 6x$.

   - Factor out the common term x: $f(x) = x(x^2 - 5x + 6)$.
   - Factorise the quadratic part: $f(x) = x(x - 2)(x - 3)$.
   - Set each factor to zero: $x = 0$, $x = 2$, and $x = 3$.
   - Identify the degree as 3 (odd) and the leading coefficient as 1 (positive), so the graph goes down to the left and up to the right.

   Answer: Roots: $x = 0, 2, 3$; End behavior: down to the left, up to the right.

3. Does the polynomial $f(x) = x^3 + x$ have any roots other than $x = 0$? Explain using factorisation.

   - Factor out the common term x: $f(x) = x(x^2 + 1)$.
   - Set the factors to zero: $x = 0$ or $x^2 + 1 = 0$.
   - Analyze $x^2 + 1 = 0$: this implies $x^2 = -1$.
   - Since no real number squared is negative, $x^2 + 1$ has no real roots.

   Answer: No, the only real root is $x = 0$.
