

What calculus is and why it was invented

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Practice problems

1. A car's velocity is given by a curved graph over time. If you want to find the total distance the car traveled between $t = 0$ and $t = 10$, are you solving the tangent problem or the area problem? Explain why.
2. You have a curve $f(x)$. You calculate the slope between two points (x_1, y_1) and (x_2, y_2) . As you move x_2 closer and closer to x_1 , what specific geometric feature of the curve are you trying to find the slope of?
3. Why can't we use the standard formula $\text{Area} = \text{length} \times \text{width}$ to find the area under the curve $f(x) = x^2$ from $x = 0$ to $x = 2$?

Solutions

1. A car's velocity is given by a curved graph over time. If you want to find the total distance the car traveled between $t = 0$ and $t = 10$, are you solving the tangent problem or the area problem? Explain why.

- Distance is the accumulation of velocity over time.
- On a graph of velocity versus time, the total distance is represented by the area between the curve and the time axis.
- Therefore, this is the area problem.

Answer: The area problem.

2. You have a curve $f(x)$. You calculate the slope between two points (x_1, y_1) and (x_2, y_2) . As you move x_2 closer and closer to x_1 , what specific geometric feature of the curve are you trying to find the slope of?

- The slope between two points is a secant line.
- As the points merge into one, the line no longer crosses the curve but just touches it.
- A line that touches a curve at a single point is called a tangent line.

Answer: The tangent line.

3. Why can't we use the standard formula $\text{Area} = \text{length} \times \text{width}$ to find the area under the curve $f(x) = x^2$ from $x = 0$ to $x = 2$?

- The formula $\text{length} \times \text{width}$ only works for rectangles.
- A rectangle has a constant height.
- The curve $f(x) = x^2$ has a height that changes at every single point along the x axis.
- Because the height is not constant, there is no single 'width' or 'length' that describes the whole shape.

Answer: Because the height of the shape is constantly changing, so it is not a rectangle.